A rheological model for the description of boulder impacts on granular strata

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This paper considers the mechanics of rock boulders impacting on granular strata. The study is aimed at improving the numerical methods devoted both to describing boulder trajectories along slopes and designing preventive structures in mountain regions. The problem is analysed by using a simplified approach inspired by the lumped mass method. This is based on (a) the macro-element concept, (b) the definition of generalised stresses and displacements, and (c) delayed plasticity theory. Both vertical and inclined impacts on horizontal strata and vertical impacts on inclined slopes are discussed. The numerical results are obtained by using a finite difference numerical discretisation to integrate, over time, a coupled system of two-dimensional differential equations. Most of the input data requested by the model are basic and geotechnically meaningful, and comparison of the numerical results with the experimental data seems to be quite promising.

KEYWORDS: elasticity; model tests; plasticity; sands; settlement; slopes

INTRODUCTION
Nowadays, because of the rapid and dramatic population of large areas of the globe, the attention paid to the safety of structures and lifelines in mountain regions is increasing everywhere. Consequently, the study of rock fall phenomena is quite important. As many uncertainties affect the problem definition in all its phases (the detachment mechanisms, the fall trajectory and the impact on structures and/or shelters), this is usually tackled statistically. Meanwhile the approaches employed are often overly approximate, because more detailed analyses would be too expensive and time consuming.

In particular, as far as the rock fall trajectory is concerned, geotechnical engineers and geologists usually solve the problem by following the mass lumped approach (Bozzolo & Pamini, 1986). In the commercial codes based on this method, the impact of a falling boulder on a rock slope, or on a stratum of soil, is described by means of appropriately defined restitution coefficients. These coefficients are used to quantify the reduction in kinetic energy of the boulder upon impact and modify its trajectory. Even though the available experimental data are continuously increasing, it is quite difficult to determine the appropriate restitution coefficient to use as a consequence of the large number of factors that influence the phenomenon. These factors include the boulder shape and the presence of vegetation along the slope.

In the design of shelters or geo-reinforced embankments, more sophisticated numerical codes are often employed, and the impact phenomenon is simulated by using either finite element numerical codes (FEM) or the discrete element method (DEM) (Cundall, 1971; Cundall & Strack, 1979). Nevertheless, because of the complexity of the problem, both the feasibility of the analysis and the reliability of the results are quite uncertain. When such a problem is analysed numerically, large deformations and rapid loading, as well as dynamic actions, must be considered.

According to the classical theories of dynamic penetration, which are commonly developed in the ballistic literature with reference to projectiles (Taylor et al., 1991), the net resistant force exerted by the soil on the penetrating object can be interpreted (Boguslavski et al., 1996) as the addition of three terms: the dissipative, the dynamic and the static components. The dissipative component is assumed to depend on losses due to viscosity and wave generation in the granular medium. Recently, the experiments of Jaeger & Nagel (1992) clearly showed that these dissipative components can be neglected in granular media. By contrast, the small-scale experimental test results of Sedov (1959) showed that the dynamic term is dominant with respect to the others even at low velocities, and depends on the projectile velocity, whereas the static component is a function of the projectile displacement.

In this paper, an alternative approach to reproduce the boulder impact phenomenon will be presented, but it will be shown that similar conclusions will be obtained at least as far as the dissipative and the dynamic terms are concerned. The dependence of the static component on the displacement, which is associated with large displacements and second-order effects, will be disregarded.

The aim of this paper is to introduce a model for interpreting and numerically simulating the impact phenomenon, considering specifically the response of a homogenous stratum of granular soil subject to the impact of a...
spherical rigid boulder. The input data will consist of the
impact velocity of the boulder, its size, and the geotechnical
parameters characterising the soil stratum; the output in-
formation includes the boulder displacement, the dynamic load
on the soil stratum, and the exit velocity vector of the
boulder.

In order to develop this rheological model, the macro-
element concept (Butterfield & Ticof, 1979; Butterfield,
1980; Georgiadis & Butterfield, 1988; Nova & Montrasio,
1991; Nova & di Prisco, 2003) was used. This formulation
was originally used to interpret the mechanical response of
rigid footings placed on homogeneous sand strata under
static inclined and eccentric loads. This model allows not
only vertical impacts on horizontal strata but even, more
generally, inclined impacts on variously inclined slopes to be
taken into consideration. The model was developed to facil-
itate implementation in lumped mass numerical codes
(Agliardi & Crosta, 2003), or for evaluating impact actions
on shelters and/or artificial tunnels. The first part of the
paper is devoted to describing the constitutive relationships,
and the second part of the paper considers the model
validation by using some experimental test results taken
from literature.

THE CONSTITUTIVE MODEL

When a rock boulder impacts on a soil stratum, a very
complex mechanical phenomenon takes place: this is due
mainly to the impact energy and to the dramatic difference
in stiffness of the two contacting materials.

The main assumptions of the BIMPAM (Boulder IMPAct
Model), as developed in this paper, are as follows.

(a) The boulder is spherical.
(b) The boulder is rigid.
(c) The boulder explosion is neglected.
(d) The spin rate of the boulder is negligible during the
impact.
(e) The soil is cohesionless.
(f) The slope geometry is described by a plane with
inclination \( \omega \).
(g) The impacting boulder velocity vector \( \mathbf{u}_B \), and the
unit vectors normal \( \mathbf{n} \) and tangential to the slope \( \mathbf{t} \)
belong to the same plane (Fig. 1), where

\[
\mathbf{t} = \left\{ \sin \omega, \cos \omega \right\}, \quad \mathbf{n} = \left\{ -\sin \omega, \cos \omega \right\} \quad (1)
\]

In order to clarify the theoretical approach chosen by the
authors, a one-dimensional sketch of the rheological model
used here is shown in Fig. 2. This one-dimensional represen-
tation is theoretically meaningful only when a vertical
impact on a horizontal isotropic or transversely isotropic
sand stratum is taken into consideration; nevertheless in this
case, as will be clarified below, the plastic slider is never
activated.

In the contact model, an elastic spring, a viscous dashpot,
a viscoplastic slider and a mass are combined as shown in
Fig. 2. The elastic spring and the viscous damper represent
the mechanical response of the deeper part of the stratum.
In particular, the viscous damper takes into account the
energy dissipated both by the propagation of waves within
the infinite elastic half space and by the irreversible response
of the soil belonging to the far field.

The viscoplastic slider simulates the penetration (Fig. 3)
taking place within the soil mass, which can be characterised
either by a punching or by a generalised failure mechanism,
whereas the standard ideal plastic slider simulates the super-
ficial sliding.

Imposing the dynamic equilibrium condition on the rheo-
logical model of Fig. 2 along the \( \omega \)-rotated axes \( \left( n-t \right) \) of
Fig. 1, the following differential equation system can be
derived.

\[
\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}(\dot{\mathbf{u}} - \dot{\mathbf{u}}^{VP} - \dot{\mathbf{u}}^{PL}) + \mathbf{K}(\mathbf{u} - \mathbf{u}^{VP} - \mathbf{u}^{PL}) = \mathbf{b} \quad (2)
\]

where

\[
\mathbf{u} = \begin{bmatrix} \mathbf{u}_N \\ \mathbf{u}_T \end{bmatrix} \quad (3a)
\]

is the displacement vector in the rotated frame of reference
\( \left( n-t \right) \), \( \dot{\mathbf{u}} \) is the associated displacement rate vector, and \( \ddot{\mathbf{u}} \)
the acceleration vector; \( \mathbf{M} \), \( \mathbf{C} \), \( \mathbf{K} \) are the mass, the viscous and
the elastic stiffness matrices respectively; \( \mathbf{b} \) is the vector of

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**Fig. 1. Definition of the planar geometry of the problem**

**Fig. 2. Schematic one-dimensional representation of the BIM-
PAM model**

**Fig. 3. Equivalent circular \( 2R \) wide rigid shallow foundation**
known constants; and \( \dot{u}^{VP} \) and \( \dot{u}^{PL} \) are respectively the viscoplastic and the plastic displacement rate vectors, being
\[
\dot{u}^{VP} = \int_0^t \dot{u}^{VP}(t) \cdot dt
\]
and
\[
\dot{u}^{PL} = \int_0^t \dot{u}^{PL}(t) \cdot dt
\]
The mass matrix \( M \) and the vector \( b \) are defined as follows.
\[
M = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \quad \text{(3b)}
\]
\[
b = mg \mathbf{t} \quad \text{(3c)}
\]
where \( g \) is the acceleration due to gravity and \( m \) is the boulder mass. The remaining terms in equation (2)—that is, the viscoplastic and the plastic displacement rate vectors and the matrices \( C \) and \( K \)—will be defined in subsequent sections of the paper.

The viscoplastic slider

The viscoplastic slider constitutive relationship introduced here is based on the plastic model conceived by Nova & Montrasio (1991) with reference to rigid footings subject to inclined and eccentric loads. This approach allows the authors to take into consideration the dependence of the mechanical response of the system on the direction of the boulder trajectory during the impact. This model is characterised by isotropic strain-hardening and by a non-associated flow rule. When the penetration of the boulder is quasi-static—that is, the boulder velocity is negligible and the increase in the impact forces is very slow—the mechanically response of the soil coincides with what was foreseen by Nova & Montrasio (1991).

Yield function and plastic potential. The yield function formulation follows the Nova & Montrasio (1991) model, but it was necessary to extend its definition to include inclined planes
\[
f = f(h, \xi, \rho_c) = h^2 - \xi^2 \left( 1 - \frac{\xi}{\rho_c} \right)^{2\beta} = 0 \quad \text{(4)}
\]
where the non-dimensional loading variables \( \xi \) and \( h \) are defined as follows:
\[
\mathbf{I} = \begin{bmatrix} \xi \\ h \end{bmatrix} = \frac{1}{V_{\text{MAX}}} \begin{bmatrix} I_N \\ I_l/\mu \end{bmatrix} \quad \text{(5a)}
\]
\[
\mathbf{I} = \begin{bmatrix} I_N \\ I_l/\mu \end{bmatrix} = \mathbf{R} \begin{bmatrix} I_N \\ I_l \end{bmatrix} \quad \text{(5b)}
\]
\[
\mathbf{R} = \begin{bmatrix} \cos \psi & -\sin \psi \\ \sin \psi & \cos \psi \end{bmatrix} \quad \text{(5c)}
\]
and the vector \( \mathbf{l} \) represents the loading in the rotated frame of reference \( (n-t) \)
\[
\mathbf{l} = \begin{bmatrix} I_N \\ I_l \end{bmatrix} = mg \mathbf{t} - m \mathbf{\dot{u}} \quad \text{(5d)}
\]
The term \( mg \mathbf{t} \) in equation (5d) is constant, and the term \( m \mathbf{\dot{u}} \) represents the inertial force vector. This implies that the yield function \( f \) depends non-linearly on the acceleration displacement vector \( \mathbf{\dot{u}} \), as considered further in the discussion on model validation presented below.

In equation (4), \( \rho_c \) is the non-dimensional hardening variable (Fig. 4(a)), \( \beta \) and \( \mu \) are constitutive parameters that describe the yield and the failure locus shape, and the rotation \( \psi \) of the yield locus is a function of the slope inclination \( \omega \) and is assumed to be positive if clockwise.

As was suggested by Nova & Montrasio (1991), \( \mu = \tan \delta \), where \( \delta \) stands for the friction angle of the interface between the rigid footing and the soil stratum, and \( \beta \) can be assumed to be constant and equal to 0.95.

\( V_{\text{MAX}} \) (equation (5a)) is the bearing capacity of the shallow circular foundation \( 2R \) wide, where \( R \) is the sphere radius (Fig. 3). \( V_{\text{MAX}} \) can be evaluated, for instance, by means of the Brinch Hansen formula (Brinch Hansen, 1970) as
\[
V_{\text{MAX}} = q_{\text{lim}} \pi R^2 = \left( \frac{1}{2} R \gamma s_y N_c \right) \pi R^2 \quad \text{(6)}
\]
where \( \gamma \) is the material unit weight, \( s_y \) is the shape coefficient, and \( N_c \) is the bearing capacity factor, which depends on the internal friction angle \( \Phi' \).

In equation (6), for the sake of simplicity, the dependence of \( V_{\text{MAX}} \) on the footing embedment is neglected. For this reason, the present version of the model cannot be employed to simulate projectile and/or bomb penetration in soils.

The tensor \( \mathbf{R} \) (equations (5)) defines the rotation of the yield locus as well as that of the failure locus associated with the slope inclination (Fig. 4(b)). A theoretical justification of this rotation has been already given in di Prisco et al. (2004). The evaluation of \( \psi \) can be done either numerically, by performing elasto-plastic FEM analyses, or empirically, by employing the standard Brinch Hansen (1970) formula. The authors used the former method, and observed that it is possible to fit the numerical data satisfactorily for inclined sand strata by assuming \( \psi = \omega/4 \).

Equations (4) and (5) define the yield function in a two-dimensional plane; by analogy, the following dimensionless definition has been chosen for the plastic potential (Nova, 1988).

![Fig. 4. (a) Representation of yield loci and failure locus for horizontal plane case; (b) representation of yield locus rotation for inclined plane case](image-url)
\[ g(\hat{\mathbf{I}}, \rho_G) = 2\beta \ln \left( \frac{\xi}{\rho_0} \right) + \left( \frac{h}{\xi} \right)^{2\beta} = 0 \]  
(7a)

from which

\[ \frac{\partial g}{\partial \hat{\mathbf{I}}} = \left[ \begin{array}{c} \frac{\partial g}{\partial \varepsilon} \\ \frac{\partial g}{\partial h} \end{array} \right] \]  = \left[ \begin{array}{c} 2\beta \left( \frac{h}{\xi} \right)^{2\beta - 1} \\ 2\beta \left( \frac{h}{\xi} \right)^{2\beta} \end{array} \right] \]  
(7b)

where \( \rho_G \) is a parameter that describes the evolution of \( g \) but does not influence the constitutive relationship.

*The flow rule.* A viscoplastic approach according to Perzyna’s (1963) theory is proposed in order to reproduce fast loading. This means that the consistency rule is disregarded, and the image point of the generalised state of stress can belong to the yield locus, and can even appear outside of it. The viscous nucleus \( \Phi(s^{-1}) \) has been assumed to depend on overstress (Eisenberg & Yen, 1981) through a suitable mapping rule,

\[ \mathbf{u}^{VP} = \left( \begin{array}{c} \mathbf{u}_{1}^{VP} \\ \mathbf{u}_{2}^{VP} \end{array} \right) = 2R \left( \begin{array}{c} u_{1}^{VP} \\ u_{2}^{VP} \end{array} \right) = 2R \mathbf{u}(d, \xi) \frac{\partial g}{\partial \hat{\mathbf{I}}} \]  
(8)

where \( d \) is a scalar measure of the overstress, which depends on the yield function definition and will be defined below. Equation (8) allows the irreversible displacements to be calculated once the viscous nucleus, the plastic potential \( g \), the yield function \( f \) and the variable \( d \) are appropriately introduced.

The variable \( d \) is assumed to coincide with the distance of the image point of the generalised state of stress \( P \) in the \( \hat{\mathbf{I}} \) plane from the yield locus according to the radial mapping rule (Fig. 5). When \( d \) is positive, point \( P \) is external to the yield locus whereas, when \( d \) is negative, \( P \) lies within the yield locus

\[ d = \left( \frac{\xi - \rho_0}{1 - |m^*|/h} \right) \sqrt{1 + m^*}^2 \]  
(9)

where \( m^* = h/\xi \). As is evident from Fig. 5, the condition \( |m^*| = 1 \) defines the tangent to the yield locus at the origin of the axes.

The viscous nucleus (Fig. 6) dramatically influences the mechanical response of the model. The analytical definition chosen by the authors is characterised by three distinct functions

**Fig. 5.** Mapping rule and definition of distance \( d \)

**Fig. 6.** Viscous nucleus definition

\[ \Phi(d, \xi) = \xi \left[ 2\gamma \sqrt{d + \Delta_1}^{0.5} + (c_V - 2\gamma \sqrt{\Delta_1}) \right] \]  
if \( d > 0 \)

\[ \Phi(d, \xi) = \xi \frac{c_V}{\Delta_2} \]  
if \( - \Delta_2 \leq d < 0 \)

\[ \Phi(d, \xi) = 0 \]  
if \( d < - \Delta_2 \)

where \( \gamma \) and \( c_V \) are dimensional \( (s^{-1}) \) constitutive parameters governing the evolution rate; and \( \Delta_1 \) and \( \Delta_2 \) are dimensionless constitutive parameters (Fig. 6) \( (\gamma \sqrt{\Delta_1}) \) represents the inclination of the tangent straight line to the \( \Phi(d)/\xi \) function evaluated for \( d = 0 \) (Fig. 6), \( c_V \) \( (s^{-1}) \) its intercept, and \( \Delta_2 \) defines the boundary of the elastic locus. The value of the parameter \( \gamma \) is associated with the characteristic time of the material, but, as the viscous nucleus definition is not linear, does not coincide with \( \gamma \).

From both the flow rule and the viscous nucleus definitions (equations (8), (9) and (10)), the viscoplastic displacement rate vector is derived. This latter will depend non-linearly on the current total acceleration vector \( \mathbf{a} \) and this will require an iterative numerical algorithm for the integration of the mathematical system, as discussed below.

When \( \omega = \theta_{IN} = 0 \) (that is when a vertical impact on a horizontal stratum is taken into consideration), in Fig. 5 both points \( P \) and \( P_1 \) belong to the \( \xi \) axis. According to the experimental observations on penetrating projectiles, the distance \( \Phi_{P1P} \) of Fig. 5 represents the dynamic component of the impact force, and the distance \( \Phi_{P} \) the static component; the dissipative term is neglected, because the viscous damper is put in series to the viscous slider.

*The hardening rule.* The hardening variable \( \rho_C \) has been introduced in equation (4) to describe the evolution of the yield locus. Different yield loci, corresponding to the case of a horizontal soil stratum, are represented in Fig. 4(a): they differ only by the \( \rho_C \) value. The hardening rule chosen for \( \rho_C \) is very close to that proposed by Nova & Montrasio (1991)

\[ \rho_C(q) = (1 - \rho_C) \frac{R_0}{V_{MAX}} \left[ \eta + \frac{\alpha}{\mu} |\dot{\varepsilon}| \right] \]  
(11)

where
\[ \dot{\mathbf{q}} = \left\{ \begin{array}{c} \eta \\ \epsilon \end{array} \right\} = V_{\text{MAX}} \cdot \mathbf{R} \cdot \mathbf{u}_{\text{VP}} \] (12)

according to which, when \( \rho_e = 1 \), failure occurs, because \( \rho_e = 0 \) even if \( \dot{\mathbf{q}} \) is not nil.

The term \( \alpha \) is an additional constitutive parameter characterising the statical mechanical response of the system, fixed and equal to 2.83 (Nova & Montrasio, 1997), while, following Nova and Montrasio’s recommendations,

\[ R_0 = \frac{30D_s V_{\text{MAX}}}{B} \] (13)

where \( B \) represents the equivalent foundation boulder diameter (Fig. 3). The contact area between the penetrating body and the ground was approximated by its planar cross-section area increasing with the normal displacement \( u_N \)

\[ B = B_0 + 2\sqrt{u_N(2R - u_N)} \] (14)

where \( B_0 \) is a fixed parameter assumed to be equal to 0.0001 m, to assign a very small but finite value for the initial viscoplastic stiffness. Conversely to what was defined in the Nova model, \( R_0 \) is not constant but varies with time; consequently, the initial plastic compliance is very large and decreases with the normal displacement \( u_N \).

**The elastic spring**

According to the Hertz theory (Mindlin & Deresiewicz, 1953), a non-linear elastic relationship for the spring was chosen. This means that the coefficients of the elastic uncoupled stiffness matrix (equation (2)) written below,

\[ K = \begin{bmatrix} k_N & 0 \\ 0 & k_T \end{bmatrix} \] (15)

depend on the normal force as follows.

\[ k_N = k_{N0} + \alpha_k l_N \] (16)

where \( \alpha_k \) and \( k_{N0} \) are constitutive parameters. In particular,

\[ k_{N0} = \frac{4GB}{1 - \nu} \] (17)

where \( G \) is an average value for the shear modulus, whose calibration is described in Appendix 1; \( \nu \) is Poisson’s ratio, and \( \alpha_k \) (m\(^{-1}\)) is a parameter describing the dependence of the elastic stiffness on the normal force. Finally, \( k_T \) (equation (15)) has been defined as follows.

\[ k_T = k_{T0} s(B) \] (18)

where \( s(B) \) is a function describing the dependence of the ratio between the tangential and the normal stiffness on the boulder mark size. A geotechnical definition for \( s(B) \) is suggested in Appendix 2.

**The viscous damper**

The viscous damper introduced in the rheological model (Fig. 2) takes into account the damping effect due to the diffusion of the elastic waves in an infinite elastic stratum. According to Sieffert & Cevaer (1991), the viscous matrix, for a circular footing, can be expressed as follows.

\[ \mathbf{C} = \begin{bmatrix} c_N & 0 \\ 0 & c_T \end{bmatrix} \] (19)

where \( c_N = k_N \frac{B}{\sqrt{G/\rho}} \eta_N \) \hspace{1cm} (20a)

\[ c_T = k_T \frac{B}{\sqrt{G/\rho}} \eta_T \] (20b)

where \( \rho \) is the soil density, and \( \eta_N \) and \( \eta_T \) are two viscous constitutive parameters. Sieffert & Cevaer’s (1991) recommendations can be adopted for the calibration of \( \eta_N \) and \( \eta_T \).

**The plastic mechanism**

When the condition \( |t_f|/l_B = \tan \delta \) is fulfilled (Fig. 4(b)), superficial sliding takes place and an internal penetration failure mechanism disappears. To describe this alternative failure mechanism, the perfectly plastic slider of Fig. 2 is introduced. When a horizontal plane is considered, the straight lines in the \( h - \xi \) plane (Fig. 4(b)) corresponding to the condition \( |t_f|/l_B = \tan \delta \) coincide with the tangential lines to the yield loci \( (m^*) = 1 \). When an inclined slope is taken into consideration, the tangential lines rotate, whereas the former ones do not.

When \( |t_f|/l_B = \tan \delta \), \( \mathbf{u}_{\text{VP}} \neq 0 \). The mechanical response of the plastic slider is governed by the following equations.

\[ \mathbf{u}_{\text{PL}} = \Lambda \frac{\partial g}{\partial t} B \] (21)

\[ \mathbf{f}_{\text{PL}} \cdot \Lambda = 0 \]

\[ \mathbf{f}_{\text{PL}} \cdot \Lambda = 0 \]

where \( f_{\text{PL}} \) is the yield function associated with the plastic slider, coinciding with the failure locus, here expressed as follows

\[ f_{\text{PL}} = |t_f|/l_B - \tan \delta = 0 \] (22)

\( \Lambda \) is the dimensionless plastic multiplier and \( g \) is the plastic potential, whose dimensionless gradient is assumed to be

\[ \frac{\partial g}{\partial t} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \] (23)

As is evident from equation (21), in the normal direction plastic displacements do not occur, and the first equation of the system (equation (2)) is not modified. However, if we consider the balance equation along the tangential axis, we obtain in this case

\[ -\mu u_N + t_f = g(\sin \omega - \mu \cos \omega) \] (24)

The differential equations (2) can be simplified and the matrices can be redefined as follows.

\[ \mathbf{M} = \begin{bmatrix} m & 0 \\ -\mu m & m \end{bmatrix}, \mathbf{K} = \begin{bmatrix} k_N & 0 \\ 0 & 0 \end{bmatrix} \]

\[ \mathbf{C} = \begin{bmatrix} c_N & 0 \\ 0 & c_T \end{bmatrix}, \mathbf{b} = mg t - mg \begin{bmatrix} 1 \\ \mu \cos \omega \end{bmatrix} \] (25)

The signs of both terms \( M_{21} \) and \( b_2 \) are fixed because, neither the normal force nor the tangential velocity change sign during the impact event. When the normal force becomes zero, boulder detachment takes place and the numerical algorithm is interrupted.

**MODEL VALIDATION**

An explicit finite difference time integration approach was chosen to solve the two-dimensional differential equation
Impact on dense sand strata

In this section the experimental results obtained by Labiouse et al. (1994) by performing small-scale tests are numerically reproduced. The experiments consider boulders with a circular cross-section, as illustrated in Fig. 8(d). The boulders had masses of 100, 500 and 1000 kg; they fell from a maximum height of 10 m vertically, and impacted on a reinforced concrete slab. Further details are given in Labiouse et al. (1994).

The experimental data (Labiouse et al., 1994) for a 100 kg boulder (Table 1) (falling height = 10 m) impacting on a 50 cm thick dense sand stratum are compared with the BIMPAM numerical simulations in Fig. 8. In this figure the boulder used experimentally is superimposed on the ideal spherical boulder whose impact is numerically simulated. The model seems to be capable of capturing the mechanical response of the system, by reproducing both the maximum vertical force applied to the soil stratum and the vertical displacement of the boulder in a satisfactory way. The viscous nucleus definition and the elastic stiffness play a fundamental role in the model response, whereas, by contrast, the viscous damper does not seem to affect the numerical solution, and can be disregarded.

Referring to Table 2, within the model some parameters are fixed and their calibration is not requested ($\alpha$, $\beta$, $C_V$, $\Delta_1$ and $\Delta_2$), because they are independent of the relative density of the soil ($D_r$). Therefore only four parameters need be introduced: $D_c$, $\gamma$, $\Phi^r$, $\gamma_N$. This last is the only parameter requiring calibration. $D_c$, $\gamma$ and $\Phi^r$ are known input data, and all of the remaining parameters in Table 2 are calculated within the numerical algorithm from the previous ones.

The statically evaluated maximum vertical load ($V_{\text{MAX}}$) is significantly smaller than the maximum vertical force recorded during the impact. This means that viscoplastic effects play a dominant role during the impact, and that the maximum experimental recorded force value is not well approximated by either the elastic or the plastic approaches. Fig. 9 shows the comparison between the elastic response, which has been obtained by depressing $\gamma_N$, and the plastic response, which has been obtained by increasing $\gamma_N$ abruptly. In the former case the dynamic force is severely overestimated, whereas in the latter it is dramatically underestimated.

After calibrating the constitutive parameters using one falling height and one boulder mass, the model has been validated by comparing the BIMPAM numerical simulations with the experimental test results in all of the other cases. To summarise the experimental data, Labiouse et al. (1994)
Boulder radius, \( R \) (lines): the maximum force \( F_{\text{MAX}} \) the interpolation lines obtained by means of equation (26) is plotted against the falling height \( H \) for three different boulder masses (100, 500, 1000 kg). The agreement between the model predictions and the experimental interpolations seem to be satisfactory: the dependence of \( F_{\text{MAX}} \) on the falling height is captured quite well, whereas by increasing the boulder mass the model seems to underestimate the value of \( F_{\text{MAX}} \). The extension of these numerical results to real impacts of rock boulders should be at any rate checked, because all the experimental impacts simulated are characterised by energetic contents that are very small compared with those associated with real rock fall events occurring in Alpine regions.

In all the cases considered above, the system analysed is one-dimensional because the stratum is horizontal and the inner velocity is vertical. The exit velocity is therefore approximately nil, and a complete energy dissipation is experimentally observed and numerically simulated. The boulder trajectory was not reported as it was meaningless. On the contrary, when either the stratum or the boulder path is inclined, the system response must be illustrated by means of both the kinematic trajectories and the loading paths. Results for inclined impacts on a horizontal dense sand stratum and vertical impacts on an inclined slope are shown in Figs 11 and 12 respectively. From Fig. 11, because the loading vector \( \mathbf{l} \) is strictly linked to the acceleration vector, we can infer that, even for very inclined velocity vectors \( (\theta = 30^\circ) \), the normal force \( (l_N) \) is dominant with respect to the tangential force \( (l_T) \), and the decrease in the normal

\[
F_{\text{MAX}} = 1.765 M_E^{2/5} R^{1/5} W^{3/5} H^{3/5}
\]  

(26)

where \( M_E \) is the stiffness modulus obtained by means of plate tests (kPa), \( H \) is the falling height (m), and \( R \) (m) and \( W \) (kN) are respectively the boulder radius and its weight. In Fig. 10 the numerical results (dots) are then compared with the interpolation lines obtained by means of equation (26) (lines): the maximum force \( F_{\text{MAX}} \) is plotted against the falling height for three different boulder masses (100, 500 and 1000 kg). The agreement between the model predictions and the experimental interpolations seem to be satisfactory: the dependence of \( F_{\text{MAX}} \) on the falling height is captured quite well, whereas by increasing the boulder mass the model seems to underestimate the value of \( F_{\text{MAX}} \). The extension of these numerical results to real impacts of rock boulders should be at any rate checked, because all the experimental impacts simulated are characterised by energetic contents that are very small compared with those associated with real rock fall events occurring in Alpine regions.

In all the cases considered above, the system analysed is one-dimensional because the stratum is horizontal and the inner velocity is vertical. The exit velocity is therefore approximately nil, and a complete energy dissipation is experimentally observed and numerically simulated. The boulder trajectory was not reported as it was meaningless. On the contrary, when either the stratum or the boulder path is inclined, the system response must be illustrated by means of both the kinematic trajectories and the loading paths. Results for inclined impacts on a horizontal dense sand stratum and vertical impacts on an inclined slope are shown in Figs 11 and 12 respectively. From Fig. 11, because the loading vector \( \mathbf{l} \) is strictly linked to the acceleration vector, we can infer that, even for very inclined velocity vectors \( (\theta = 30^\circ) \), the normal force \( (l_N) \) is dominant with respect to the tangential force \( (l_T) \), and the decrease in the normal

\[
F_{\text{MAX}} = 1.765 M_E^{2/5} R^{1/5} W^{3/5} H^{3/5}
\]  

(26)

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force associated with an increase in $\dot{\gamma}_V$ is not severe. The numerical simulations presented in Figs 11 and 12 are summarised in Fig. 13 by taking the modulus ($|\dot{\gamma}_V|$) of maximum force into consideration. In both cases, the angles $\Theta$ and $\dot{\gamma}_V$ do not influence the maximum force significantly.

When an impact on an inclined slope and/or an inclined boulder trajectory is considered, the generalised stress path can help in interpreting the system response. According to the relative inclination between the normal to the soil stratum and the velocity vector, superficial sliding after an initial penetration failure mechanism can take place or not, whereas transition from one to the other is continuous, and the boulder is likely to rebound.

A classical approach to describe empirically the response of a dissipative cushion to a dynamic impact relies on using restitution coefficients defined as:

$$\varepsilon = \frac{v_{final} - v_{initial}}{v_{initial}}$$

where $v_{final}$ and $v_{initial}$ are the final and initial velocities, respectively.

### Table 2. Geotechnical characterisation for the soil (Lausanne test series, dense sand)

<table>
<thead>
<tr>
<th>Fixed parameters</th>
<th>Input data</th>
<th>Dependent parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative density, $D_r$</td>
<td>90</td>
<td>Janbu elastic parameters, $K$, $n$</td>
<td>800, 0.40</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Viscous damping parameter, $\eta$</td>
<td>0.85, 0.58</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Poisson’s ratio, $\nu$</td>
<td>0.25</td>
</tr>
<tr>
<td>Soil unit weight, $\gamma$</td>
<td>18</td>
<td>Rock/soil interface friction angle, $\delta$</td>
<td>30</td>
</tr>
<tr>
<td>Internal friction angle, $\Phi$</td>
<td>40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dependence of elastic stiffness on load, $\alpha_K$</td>
<td>0</td>
<td>Viscous damping parameter, $\beta$</td>
<td>0.95</td>
</tr>
<tr>
<td>Hardening parameter, $\alpha$ (-)</td>
<td></td>
<td></td>
<td>2.53</td>
</tr>
<tr>
<td>Viscous nucleus parameter, $\gamma_V$</td>
<td>4.32</td>
<td>Viscous nucleus parameters, $c_1$, $\Delta_1$, $\Delta_2$ (-)</td>
<td>1, 5, -1</td>
</tr>
</tbody>
</table>

Fig. 9. Comparison between mechanical responses obtained by reducing (dashed line) and increasing (continuous line) value of $\gamma_V$: (a) vertical acceleration, (b) velocity and (c) displacement against time. In the graphs the simulations of Fig. 8 are also reported (dotted line).
The constitutive model seems capable of simulating the experimental values of the impact dynamic loading from both a qualitative and a quantitative point of view; by contrast, the trajectories of the boulder during the impact are not satisfactorily reproduced (Fig. 14(d)). According to the present authors, this discrepancy is due to the large values of the vertical penetration, which make the second-order effects significant: this means that the value of $F_{\text{MAX}}$ (equation (6)) should be updated and, because the boulder sinks, superficial sliding should be inhibited.

Constitutive parameter remarks

Finally, to summarise the role played by the constitutive parameters in the model, it will be useful to stress the following points.

(a) The role of $\gamma_V$ and $\alpha_K$ is quite important; in fact, the maximum value of the force depends mostly on the viscous nucleus definition and on the material elastic stiffness. In particular, by increasing $\gamma_V$ the maximum force decreases, because the soil microstructure rearrangement becomes more rapid and the ‘pseudo-instantaneous’ stiffness decreases. In contrast, by increasing the elastic stiffness, the maximum impact force increases.

(b) The shape of the curve of acceleration against time is dominated by the definition of the function describing the viscous nucleus. In particular, the viscous nucleus shape for negative $d$ values—that is, when the generalised stress image point is within the yield locus— influences the severe change in the tangent to the curve, evident, for instance, at point P of Fig. 8(a).

CONCLUSIONS

This constitutive model is intended to be an engineering tool for describing impact events on homogeneous sand strata. It is, at the same time, a numerical and a heuristic tool. The evaluation of the impact force and the exit velocity is achieved by the numerical integration of a non-linear differential equation system with two degrees of freedom. The iterative implemented algorithm is explicit conditionally stable and its integration is rapid. If it is appropriately coupled with other numerical codes, it can facilitate the design of damping ditches and artificial tunnels.

The constitutive model is based on the macro-element concept and on the theory of delayed plasticity. The model is based on the assumption that the boulder is rigid, and the time dependence of the mechanical response of the soil stratum is essentially due to the severe fabric rearrangement induced by the boulder penetration. As a consequence, this dependence disappears when superficial sliding takes place. Hence two different failure mechanisms are introduced: a coupled viscoplastic one (which is dominant when the approach velocity is parallel to the unit vector normal to the slope) and a standard Coulomb friction mechanism (taking place when the trajectory becomes sufficiently inclined with respect to the normal to the stratum). During the evolution of time, as is evident from the numerical simulations discussed above, a transition from one failure mechanism to the other may occur.

The definition of the viscous nucleus represents a crucial point in the constitutive modelling: it significantly affects the loading–time history and, at the same time, the displacements of the boulder. Moreover, this choice severely influences the relationship between the impact energy, which depends both on the mass of the boulder and
on the falling height, and the maximum value of the impact force. Because of the coupling between the horizontal and vertical loading and kinematic components, the model seems to be capable of reproducing the impact of boulders on both horizontal and inclined slopes. In this case, the generalised stress path and the boulder trajectory analysis lead us to a better understanding of the evolution of the impact event.

Most of the model constitutive parameters are strictly linked to very common geotechnical parameters: for this reason their calibration is quite simple. Others have been calibrated on experimental results taken from literature and were kept constant. Further experimental results could make it possible to validate the model and the relative constitutive parameter calibration, even for impacts characterised by greater energetic contents.

Fig. 11. Inclined impact of a 100 kg boulder on a flat dense stratum (impact velocity 14 m/s): (a) boulder trajectory; (b) vertical and horizontal acceleration; (c) stress paths in the $\xi$–$h$ plane; (d) restitution coefficients.
Fig. 12. Vertical impact of a 100 kg boulder on an inclined dense stratum (impact velocity 14 m/s): (a) boulder trajectory; (b) vertical and horizontal acceleration; (c) stress paths in the $\xi-h$ plane; (d) restitution coefficients

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Appendix 1

In order to evaluate the average shear modulus $G$, for use in equation (17), the following equations, as proposed by Janbu (1963), were used.

$$ E = K \rho_{\text{ATM}} \left( \frac{\sigma'_C}{\rho_{\text{ATM}}} \right)^n $$ (28)

$$ \tilde{G} = \frac{E}{2(1 + \nu)} $$ (29)

where $\rho_{\text{ATM}}$ is the atmospheric pressure, and $K$ and $n$ are dimensionless constitutive parameters depending on the material relative density, according to empirical tables suggested by Janbu (1963). By assuming an oedometric
normally consolidated effective state of stress, the confining pressure \( \sigma_c \) is evaluated empirically, as follows.

\[
\sigma_c \approx 0.45 \gamma f
\]  

(30)

where \( \gamma \) is the natural unit weight of the material, and the average depth \( f \) is given by

\[
\bar{f} = u_{\text{N}} + B/2
\]  

(31)

APPENDIX 2

In order to define the function \( s(B) \) used in equation (18), the stress distribution within the soil stratum was assumed to coincide with that obtained by Poulos & Davis (1974) for a homogeneous soil, characterised by constant values of Poisson’s ratio and Young’s modulus. The applied surface stresses are assumed to act on a circular area. To evaluate \( k_N \) the loading is assumed to be purely normal, whereas to evaluate \( k_T \) purely tangential loading is assumed.

Both the normal, \( u_N \), and the tangential, \( u_T \), displacements are then evaluated by employing an approach analogous to that proposed by Schmertmann (1970), in which the soil stratum is assumed to be a stratified medium. The values \( E_i \) associated with each layer have been calculated by means of equation (28), in which the relative depth of the layer is introduced. Therefore the calculated ratio \( s(B) = k_T/k_N \) will depend on the diameter \( B \) of the loaded area, and this stems from the fact that the stress distributions associated with the two distinct loading cases are very different.

In Fig. 15 the calculated trend is compared with an analytical interpolating curve. This means that, during the impact, the ratio \( k_T/k_N \) changes as \( B \) evolves. In the numerical code the analytical interpolation is introduced, and the numerical coefficients are not constitutive parameters to be input, but rather are hidden fixed parameters.

APPENDIX 3

As anticipated, and as has been experimentally illustrated by Calvetti et al. (2004), when the impact of the boulder takes place on a loose granular stratum, the granular assembly undergoes rapid densification. This implies a state transformation of the material and, as a consequence, this should influence all the constitutive parameters of the model depending on the relative density \( D_r \).

A crucial problem consisted therefore in conceiving a convincing manner to evaluate the relative density evolution of the stratum. A satisfying solution to this problem was envisaged in assuming that the relative density \( D_r \) depends on the viscoplastic work \( (L^V = \int_{t_0}^{t} \dot{\mathbf{u}}^V \, dt) \) (Fig. 16) according to the following incremental relationship:

\[
D_r = 10^{-5} (1 - D_r) L^V
\]  

(32)

where \( \dot{L}^V \) is the total incremental viscoplastic work and \( 10^{-5} \) is a fixed scale factor, dimensionally defined as \( \text{N}^{-1} \text{m}^{-1} \).

For the sake of simplicity, only the parameter \( \gamma_V \) was assumed to depend on \( D_r \) and the following evolution rule has been introduced (Fig. 16).

\[
\gamma_V = (\gamma_{V, \text{MAX}} - \gamma_{V, \text{MIN}}) D_r + \gamma_{V, \text{MIN}}
\]  

(33)

where \( \gamma_{V, \text{MAX}} \) and \( \gamma_{V, \text{MIN}} \) are the maximum and the minimum values of \( \gamma_V \), corresponding respectively to \( D_r = 100\% \) and \( D_r = 0\% \), as listed in Table 3. Equation (33) ensures that when both the relative density is sufficiently high and the kinetic energy content of the impacting boulder is negligible, \( \gamma_V \) does not evolve.

NOTATION

\( B \) diameter of equivalent circular foundation  
\( B_0 \) initial value for \( B \)  
\( b \) known terms vector in solving system  
\( c_{N_s}, c_T \) viscous coefficient  
\( D_r \) relative density  
\( d \) overstress variable  
\( E \) Young’s modulus  
\( f \) yield locus  
\( G \) shear modulus  
\( g \) plastic potential  
\( k_N, k_T \) elastic stiffness  
\( l \) loading vector in the rotated frame  
\( \psi \)-rotated loading vector  
\( \psi \) generalised loading variables vector  
\( \mathbf{M}, \mathbf{C}, \mathbf{K} \) masses, damping and elastic stiffness matrices  
\( m \) boulder mass  
\( m^* \) slope in the \( \xi - h \) plane  
\( N_c \) bearing capacity factor  
\( q \) generalised viscoplastic strains vector  
\( q_{\text{lim}} \) bearing capacity  
\( R \) \( \psi \)-rotation tensor  
\( \bar{R} \) boulder radius  
\( R_0 \) plastic stiffness  
\( s \) shape factor  
\( \lambda(B) \) evolution law for \( k_T \)
\textbf{REFERENCES}


