Service reliability analysis of a tramway network

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ABSTRACT: A promising approach to the analysis of complex technological network systems and critical infrastructures comes from the findings of Complexity Science. The underlying viewpoint is that the stability and robustness of a network system depend on the redundant wiring of the functional web interconnecting its components. In addition to the classical topological indicators, by considering the ‘reliability distances’ among network nodes in terms of the probabilities of failure of the interconnecting links, global and local reliability efficiency indicators can be defined for use in the analysis of the robustness and vulnerability of network systems and thus for their optimal design, operation and management. This paper presents the analysis of the tramway network system of the city of Milano, Italy, whose reliability is defined with respect to the probability of timely servicing. Both topological and reliability modifications are applied to the original network configuration to analyze the corresponding changes in the service reliability efficiency of the system.

1 INTRODUCTION

Distributed systems and infrastructures constitute the backbone of modern industry and society (e.g. computer and communication systems (Aggarwal 1975, Kubat 1989, Sumad 1987), power transmission and distribution systems (Jane et al. 1993, Yeh 1998), rail and road transportation systems (Aven 1987), oil/gas systems (Aven 1987, Aven 1988)). Identifying and quantifying the vulnerabilities of such complex systems is crucial for designing the adequate protections, mitigation and emergency actions against their failures.


Along this line of thought, novel indicators of the local and global reliability characteristics of a complex network system have been recently introduced (Zio, in press) as an extension of the network efficiency measures introduced in (Latora & Marchiori 2001) and as complement to the classical topological indicators such as the characteristic path length and the clustering coefficient (Watts & Strogatz 1998). By considering the ‘reliability distances’ among network nodes in terms of the probabilities of failure of the interconnecting links, global and local reliability efficiency indicators can be defined for use in the analysis of the robustness and vulnerability of network systems and thus for their optimal design, operation and management.

This paper presents the analysis of the tramway network system of the city of Milano, Italy, whose reliability is defined with respect to the probability of timely servicing. Both topological and reliability modifications are applied to the original network configuration to analyze the corresponding changes in the service reliability efficiency of the system.

For comparison, two typical network topologies with the same number of nodes of the original tramway network have been analyzed with respect to the newly developed global and local reliability efficiency indicators: the small-world topology, characterized by high clustering and good global accessibility (Albert et al. 2000) and the scale-free topology, characterized by few connections to most nodes and only a small number of highly-connected nodes, thus making it error tolerant but also extremely fragile to attacks to the highly connected nodes (Albert et al. 2000).

The paper is structured as follows: Section 2 contains the analysis of the original topology of the Milano tramway network. A crude sensitivity analysis is reported in Section 3 on the performance of the network with respect to changes in the reliability of its links. The effects of modifications to the
network topology are also considered. A comparison to small-world and scale-free networks with the same number of nodes is offered in Section 4. Conclusions on the outcomes of the analysis are eventually drawn in Section 5.

2 ANALYSIS OF THE MILANO TRAMWAY NETWORK

2.1 System description

The tramway system of Milano is currently made up of 19 lines which cover the town area from the center to the suburbs. The system can be modeled as a stochastic, weighted, undirected, connected network in which each tram stop is transposed into a node, linked by edges representing the rail tracks connecting consecutive stops (Fig. 1).

Mathematically, the network of \( N = 407 \) nodes (hereafter also called vertices) connected by \( K = 446 \) edges (hereafter also called links or arcs) can be represented by a graph \( G(N, K) \) defined by its \( N \times N \) adjacency (connection) matrix \( \{a_{ij}\} \) whose entries are 1 if there is an edge joining node \( i \) to node \( j \) or 0, otherwise. Since the majority of nodes are linked only to the two adjacent ones representing the first neighbor stops on a line, the adjacency matrix of the tramway network is rather sparse.

The stochastic behavior of the tram service is modeled by introducing the probability \( \xi_{ij} \) that from a given stop \( i \) the tram will get to the next stop \( j \) on time, according to schedule. For the sake of simplicity, it is assumed that \( \xi_{ij} = \xi \) for the whole tram network. This is obviously a strong simplification since in reality such probability depends on several factors such as the area of the city where the link lies and the distance between the stops, to name a few. Furthermore, external events such as traffic jams, accidents and so on can lead to common cause failures in a given area, without affecting other parts of the network. For a more realistic assessment of the service probability one may resort to advanced modeling approaches such as those of queuing theory (Newell 1982).

If two stops, i.e. two nodes \( i \) and \( j \), are linked by more than one tram line, say \( m \), the probability of servicing on time the two connected nodes becomes:

\[
p_i = 1 - (1 - \xi)^m
\]  

Again, service independence between different possible lines is a simplifying assumption.

We shall refer to \( p_{ij} \) as the (service) reliability of edge \( ij \) and call service failure probability of edge \( ij \) its complement to one, \( q_{ij} = 1 - p_{ij} \). Thus, in addition to the adjacency matrix \( \{a_{ij}\} \), the additional matrix \( \{p_{ij}\} \) (or the complementary \( \{q_{ij}\} \)) is introduced to describe the tramway system.

\[
p_i = \min \left( \prod_{m=1}^{\infty} p_{mn} \right) = \min \left( \prod_{m=1}^{\infty} (1 - q_{mn}) \right)
\]

where the minimization is done with respect to all paths \( p_{ij} \) linking nodes \( i \) and \( j \) and the product extends to all the edges of each of these paths (Zio, in press).

To characterize the global and the local connectivity performances of the network, both topology and efficiency indicators have been considered. From the topological point of view, the evaluation of the characteristic path length \( L \) and the clustering coefficient \( C \) has been carried out (Watts and Strogatz 1998). The former gives a measure of the average distance (number of edges) between two generic vertices and as such it measures a global property of the network topology; on the contrary, the latter gives local information about the connectivity of the subgraph formed by each node.

From the network efficiency point of view, the focus is on the probability of ‘on time service’ between pairs of nodes \( i \) and \( j \). On the basis of both \( \{a_{ij}\} \) and \( \{p_{ij}\} \) (or the complementary \( \{q_{ij}\} \)), the matrix of the shortest (most ‘service reliable’) path lengths \( \{d_{ij}\} \) can be computed:

\[
d_i = \min \left( \prod_{m=1}^{\infty} p_{mn} \right) = \min \left( \prod_{m=1}^{\infty} (1 - q_{mn}) \right)
\]
Averaging the efficiency of the local neighborhoods

The results of the topology indicators $L$ and $C$ (independent of $\xi$) and of the service efficiencies are reported in the second column of Table 1, for the (optimistic) base case of $\xi = 0.9$. The predominant series structure of the network is responsible for the large number of sparse subgraphs around the nodes, a phenomenon which leads to the small values of the average clustering coefficient and local efficiency. At the same time, the absence of shortcuts between tram lines leads to poor global accessibility, as demonstrated by the large characteristic path length and small global efficiency (on average, one must travel 20 stops to go from one node to another). In summary, the global and local connectivities of the tramway network are very poor, due to the mainly sequential structure of the system. As a result, if a node is disconnected many nodes become no longer connected to each other. Thus, the network does not present the desirable small world characteristics of good global and local connectivity.

### 3 SENSITIVITY ANALYSIS

A sensitivity analysis with respect to the probability $\xi$ has been carried out with the aim of studying how a change in service efficiency between two consecutives stops affects the performance of the entire system. Figures 2 and 3 show the behavior of the global and local efficiencies for values of $\xi$ in the interval $[0.2, 0.9]$. For all values of $\xi$ in $[0.2, 0.9]$, the values of both the global and the local efficiencies are rather small in an absolute sense. Further, there is a change of only a factor 3–5 for values of $\xi$ in $[0.2, 0.9]$. This relatively small variation in the service reliability efficiency at both the global and local scales is a confirmation that the scarce efficiency of the network is due to its topological connectivity which is quite sparse because many nodes are linked only to the previous and following ones along the shared service line without shortcuts to other lines. Note that obviously the additional information on the service efficiency sensitivity cannot be conveyed by the topological indicators $L$ and $C$ which are not affected by changes in $\xi$ as these do not modify the system structure.

The analysis of the properties of the network system has been carried out further by studying the effects of implementing changes in the structure itself. First, the

| Table 1. Topological indicators L, C and service efficiencies $\xi$ |
|-----------------|-----------------|-----------------|
|                 | I               | II              | III             |
| L               | 20              | 16              | 17              |
| C               | $2.62 \cdot 10^{-2}$ | $2.66 \cdot 10^{-2}$ | $2.62 \cdot 10^{-2}$ |
| $E_{\text{glob}}(G)$ | $6.75 \cdot 10^{-2}$ | $7.81 \cdot 10^{-2}$ | $7.93 \cdot 10^{-2}$ |
| $E_{\text{loc}}(G)$ | $1.83 \cdot 10^{-2}$ | $1.83 \cdot 10^{-2}$ | $1.83 \cdot 10^{-2}$ |

I: original configuration.
II: Connected terminals.
III: Most connected nodes.
Figure 2. Global efficiency $E_{\text{glob}}(G)$ versus $\xi$.

Figure 3. Local efficiency $E_{\text{loc}}(G)$ versus $\xi$.

Figure 4. The modified tramway network in which all terminals have been connected together.

Figure 5. The modified tramway network in which the six most connected nodes have been linked together.

Indeed, the new connections actually shorten some paths in the network but they do not change sensibly its structure so that the overall service efficiency is not significantly improved. On the contrary, the local properties are basically not affected at all by this new ‘suburban’ line connecting all terminals.

Similar results are obtained when linking together the 6 most connected nodes, i.e. those with connectivity degree $k = 5$ (Figure 5 and Table 1, column 4).

In general, then, creating few new links does not change substantially the properties of the tramway network which is strongly serial.

Finally, the robustness of the original network configuration to the disconnection, one by one, of the six most connected nodes ($k = 5$) has been investigated in the two extreme cases of edge service reliability $\xi = 0.2$ and $\xi = 0.9$. No significant change is shown at either the global or local scales when these nodes are removed one by one, although actually the characteristic path length goes to infinity because some nodes are no longer reachable. When all the six most connected nodes are removed, $E_{\text{glob}}(G)$ decreases from $1.9 \cdot 10^{-2}$ to $1.3 \cdot 10^{-2}$ for $\xi = 0.2$ and from $6.8 \cdot 10^{-3}$ to $4.9 \cdot 10^{-2}$ for $\xi = 0.9$, the relative change being comparable due to the fact that the modification is topological and the service reliability $\xi$ is equal for all edges of the network. Note that these variations cannot be analyzed with the topological indicator $L$ since its value goes to infinity when a node is no longer reachable in the network (in contrast, a remarkable feature of the network efficiency indicators is precisely that they apply even in case of some nodes being disconnected from the remainder of the network).

4 COMPARISON WITH SMALL WORLD AND SCALE-FREE NETWORKS

Two connected, weighted, undirected networks with the same number of nodes $N = 407$ as the original network but with small-world and scale-free topologies have
Table 2. Reliability efficiency indicators for the small-world network.

<table>
<thead>
<tr>
<th>$p_{ij}$</th>
<th>[0.0, 0.8]</th>
<th>[0.8, 0.9]</th>
<th>[0.9, 0.99]</th>
<th>[0.99, 1]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{glob}$</td>
<td>0.223</td>
<td>0.374</td>
<td>0.415</td>
<td>0.433</td>
</tr>
<tr>
<td>$E_{loc}$</td>
<td>0.261</td>
<td>0.471</td>
<td>0.523</td>
<td>0.549</td>
</tr>
</tbody>
</table>

Figure 6. Connectivity distribution for the scale-free network, $P(k) \sim k^{-1.3}$.

Table 3. Reliability efficiency indicators for the scale-free network.

<table>
<thead>
<tr>
<th>$p_{ij}$</th>
<th>[0.0, 0.8]</th>
<th>[0.8, 0.9]</th>
<th>[0.9, 0.99]</th>
<th>[0.99, 1]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{glob}$</td>
<td>0.155</td>
<td>0.297</td>
<td>0.330</td>
<td>0.346</td>
</tr>
<tr>
<td>$E_{loc}$</td>
<td>$2.53 \cdot 10^{-2}$</td>
<td>$5.33 \cdot 10^{-2}$</td>
<td>$5.92 \cdot 10^{-2}$</td>
<td>$6.23 \cdot 10^{-2}$</td>
</tr>
</tbody>
</table>

Table 4. Reliability efficiency indicators in Table 2 normalized to the [0.9, 0.99] case.

<table>
<thead>
<tr>
<th>$p_{ij}$</th>
<th>[0.0, 0.8]</th>
<th>[0.8, 0.9]</th>
<th>[0.9, 0.99]</th>
<th>[0.99, 1]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{glob}$</td>
<td>0.537</td>
<td>0.901</td>
<td>1</td>
<td>1.043</td>
</tr>
<tr>
<td>$E_{loc}$</td>
<td>0.499</td>
<td>0.900</td>
<td>1</td>
<td>1.050</td>
</tr>
</tbody>
</table>

Table 5. Reliability efficiency indicators in Table 3 normalized to the [0.9, 0.99] case.

<table>
<thead>
<tr>
<th>$p_{ij}$</th>
<th>[0.0, 0.8]</th>
<th>[0.8, 0.9]</th>
<th>[0.9, 0.99]</th>
<th>[0.99, 1]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{glob}$</td>
<td>0.470</td>
<td>0.900</td>
<td>1</td>
<td>1.048</td>
</tr>
<tr>
<td>$E_{loc}$</td>
<td>0.427</td>
<td>0.900</td>
<td>1</td>
<td>1.052</td>
</tr>
</tbody>
</table>

been analyzed with respect to their service reliability efficiency properties.

The small-world network has been created resorting to the Watts-Strogatz model (Watts and Strogatz 1998). To illustrate the onset of the small-world property, Watts & Strogatz (1998) have proposed a model to construct a class of unweighted graphs which interpolates between a regular lattice and a random graph. The edges of a regular lattice of $N$ nodes with $n$ first neighbors connected are rewired with a probability $\pi$. In our example, $n = 11$ and $\pi = 0.11$. To weigh the unweighted network, each edge connecting nodes $i$ and $j$ has been assigned a probability of successful connection (i.e., on time service) $p_{ij}$, sampled from a uniform distribution on a given interval. The resulting network of $N = 407$ nodes connected by $K = 8800$ edges shows small-world properties, bearing both high global and local reliability efficiencies. A sensitivity analysis has been performed by sampling on different intervals (Table 2). Again, since this analysis does not affect the topology of the system, the topological indicators, i.e., the characteristic path length and the clustering coefficient, are not able to convey any information in this respect.

The scale-free network has been created resorting to the Barabasi-Albert model (Barabasi et al. 1999). The essential features of this model are the growth of the network by the continuous addition of new nodes and the preferential attachment of new edges (i.e., the likelihood of connecting to a node depends on the node’s degree of connectivity). The resulting network of $N = 407$ nodes connected by $K = 2376$ edges presents a power-law connectivity distribution (Figure 6).

Accordingly, the network is characterized by a large value of the global reliability efficiency, similarly to the small-world network, but associated with a lack of local efficiency. The results of the same sensitivity analysis with respect to the values of $p_{ij}$ are reported in Table 3.

It is interesting to note, as more clearly shown in Tables 4 and 5 where the efficiencies are normalized with respect to the case of $p_{ij}$ uniform in [0.9, 0.99], that for uniform values of $p_{ij}$ in a given interval, the relative changes in the global and local efficiencies are practically the same. This behavior can be explained by observing that actually no relatively different service reliability path is created in the network since all the edges reliabilities are on average changing to the same extent. Consequently, the reliability efficiency indicators change uniformly both on the global and local scales.

Finally, with respect to the scale-free network, it is interesting to analyze its response to attacks. In this respect, it is known that this type of networks are affected by the removal of the most connected nodes whereas they are error tolerant, i.e. the ability of their nodes to communicate is unaffected by the failure of randomly chosen nodes. Figures 7 and 8 show...
5 CONCLUSIONS

Recent insights derived from the study of network systems from the point of view of Complexity Science can be exploited to study the global and local reliability characteristics associated to the service provision by a complex system or critical infrastructure.

Service reliability efficiency measures can be introduced considering the 'service reliability distances' among network nodes in terms of the probabilities of service failure of the interconnecting links. This leads to the definition of global and local reliability efficiency indicators which can complement the classical topological measures, like the characteristic path length and clustering coefficient, for the analysis of the robustness and vulnerability of network systems and thus for their optimal design, operation and management.

In this paper, the reliability efficiency indicators have been applied to the analysis of the tramway network system of the city of Milano, Italy. Each stop is transposed into a node in the network. Adjacent nodes on the same service lines are connected by edges. The network topology is characterized by a large number of nodes being connected only with the previous and subsequent ones on the line. This predominant sequential structure leads to a sparse adjacency matrix and affects the global and local connectivity properties in a negative sense. Due to this topologically sparse structure, no adequate modification of the topology or the line service reliability seems to be effective at the system level in terms of its service reliability efficiency.

Several modeling assumptions have been made, an analysis of which would be of interest for a future work including a study of the sensitivity of the indicators to the different simplifications and a comparison with other techniques.

REFERENCES


Zio, E., From Complexity Science to reliability efficiency: a new way of looking at complex network systems and critical infrastructures, accepted for publication on International Journal on Critical Infrastructures.

The network visualizations were done using the Pajek program for large network analysis: http://vlado.fmf.unilj.si/pub/networks/pajek/pajekman.htm