Vulnerability analysis of a power transmission system

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Abstract: Measures of topological interconnection and reliability efficiency are used for the analysis of the robustness against element failure and the resilience against directed attacks of network infrastructures. An application to an electrical power transmission system is given.

Keywords: Critical infrastructure, power transmission system, resilience and robustness analysis, topological and reliability indicators.

1. INTRODUCTION

Error tolerance and attack resilience of critical infrastructures depend on their redundant network configurations [1]. On the other hand, the complexity of these network systems renders their analysis quite difficult by classical methods. When examining realistic conditions of operations, one must take into account the probability of occurrence of faults and attacks in the various points of the network. To this aim, local and global reliability indicators have been introduced [2] as network efficiency measures [3] complementing the classical topological indicators, like the characteristic path length and the clustering coefficient [4]. By considering the ‘reliability distances’ among network nodes in terms of the probabilities of failure of the interconnecting links, these indicators give additional insights into the robustness and resilience of the network systems, useful for their optimal design, operation and management.

In this paper, the significance of the topological and reliability indicators is investigated by applying them on a case study regarding the characterization of the robustness and resilience properties of the transmission network system of the IEEE (Institute of Electrical and Electronic Engineers) 14 BUS (a portion of the American Electric Power System) [5]. A comparison is made with a random graph with the same number of nodes and edges, to verify whether the transmission system holds the desired small-world properties of high clustering and good global accessibility [1]. A validation of the quantitative results provided by the topological and reliability indicators is performed by Monte Carlo simulation.

The paper is structured as follows: Section 2 contains the description of the transmission network [5]; Section 3 introduces the topological analysis of the IEEE 14 BUS network system and the comparison with a random graph with the same number of nodes and edges; the reliability analysis of the IEEE 14 BUS is discussed in Section 4 and conclusions on the outcomes of the analysis are eventually drawn in Section 5.

2. TRANSMISSION NETWORK DESCRIPTION

The transmission network system considered represents a portion of the American Electric Power System and consists of 14 bus locations connected by 20 lines and transformers as shown in Figure 1 [5]. The transmission lines are at two voltages, 132 kV and 230 kV. The 230 kV system is the top part of Figure 1, with 230/132 kV tie stations at Buses 4, 5 and 7. Buses 1 and 2 are the generating units. The system has also voltage corrective devices at Buses 3, 6 and 8 (synchronous condensers).

Mathematically, the network can be represented as a graph $G(N,K)$ with $N=14$ nodes (hereafter also called vertices) connected by $K=20$ edges (hereafter also called arcs) as shown in Figure 2. The connections are defined in an $N\times N$ adjacency matrix $\{a_{ij}\}$ whose entries are 1 if there is an edge joining node $i$ to node $j$ and 0 otherwise.

The network visualizations were done using the Pajek program for large network analysis [6].

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3. TOPOLOGICAL ANALYSIS

3.1. Topological indicators

The transmission network is first analyzed from a purely topological point of view. Each link is considered having a length equal to one and thus the distance between two nodes $i$ and $j$ is represented solely by the number of edges travelled in the path from $i$ to $j$. Furthermore, since $G(N,K)$ is a connected graph, there are $N \cdot (N-1)/2$ distinct shortest paths among the $N$ nodes.

An indicator of the topology of a complex network is the distribution of the degree (or connectivity) $k_i$ of node $i$, defined as the number of edges incident with the node:

$$k_i = \sum_{j \in N} a_{ij}, \quad i = 1, 2, ..., N \tag{1}$$

The degree distribution can be synthesized by its average value:

$$\overline{K} = \frac{1}{N} \sum_{i=1}^{N} k_i \tag{2}$$
Further insights on the connection characteristics of the network are gained with the characteristic path length $L$ and the clustering coefficient $C$ [4]. The former is the average of the shortest path length $d_{ij}$ (least number of edges) between all pairs of nodes and gives a measure of the average distance between two generic vertices. The maximum value of $d_{ij}$ is called the diameter of the graph, $D$. On the contrary, the clustering coefficient gives local information on the connectivity of the subgraph formed by each node and is given by the average of the local clustering coefficients $c_i$ of the nodes $i=1,2,...,N$ over all the nodes in $G$.

Since $L$ diverges if there are disconnected components in the graph, an alternative approach is to consider the so-called global efficiency of $G$ [3]:

$$E = \frac{1}{N(N-1)} \sum_{i,j: i \neq j, i \neq j} \frac{1}{d_{ij}}$$

where $e_{ij} = 1/d_{ij}$ can be considered as the efficiency of the connection between nodes $i$ and $j$ in terms of the number of edges on the shortest path linking the two nodes. Correspondingly, the local properties of the graph $G$ can be quantified by specializing the definition of the global efficiency (3) on the subgraph $G_i$ of the neighbors of each node $i$ in the network [3],

$$E(G_i) = \frac{1}{k_i(k_i-1)} \sum_{l,m: l \neq m, l \neq m} \frac{1}{d_{lm}}$$

where the quantities $d_{lm}$ are the shortest distances between nodes $l$ and $m$ calculated on the subgraph $G_i$. Averaging the efficiency of the local neighborhoods of all nodes in the network, a measure of the network local efficiency is defined:

$$E_{loc} = \frac{1}{N} \sum_{i \in G} E(G_i)$$

### 3.2. Topological analysis of the IEEE 14 BUS network

For the IEEE 14 BUS transmission network described in Section 2, the following values of the above topological indicators are obtained:

$D = 5.000; \quad \bar{K} = 2.857$

$L = 2.374; \quad C = 0.367$

$E = 0.522; \quad E_{loc} = 0.392$

The corresponding values for a random graph with the same number of nodes and edges are:

$D = \infty; \quad \bar{K} = 2.428$

$L = \infty; \quad C = 0.167$

$E = 0.403; \quad E_{loc} = 0.167$

The disordered nature of the arrangement of the links between different nodes in the random graph has been generated by the algorithm developed by Erdos and Renyi for connecting couples of randomly selected nodes [7].

Note the values $L=\infty$ and $D=\infty$ for the random graph, due to the incompleteness of the connection web of the network: as mentioned earlier, this is a limitation of these indicators that give finite values only for fully connected networks. This limitation can be overcome by referring to the global efficiency indicator $E$ of (3) which has a finite value for any kind of network, fully connected or not [3].

A comparison of the results obtained shows that the IEEE 14 BUS is characterized by large values of both the global and local efficiencies; this is typical of small world networks, with good robustness properties [4].

### 4. RELIABILITY EFFICIENCY ANALYSIS

#### 4.1. Reliability efficiency indicators

To further delve into the properties of the transmission network system, the analysis is extended to adopting the formalism of weighted networks [3] for accounting the connection reliabilities [2].
For the electrical power transmission network, the focus is on the reliability $p_{ij}$ of power flow between pairs of nodes $i$ and $j$. We refer to this quantity as the reliability of edge $ij$, defined as:

$$p_{ij} = e^{-\lambda_{ij} T},$$

where $\lambda_{ij}$ is the failure rate of edge $ij$ linking nodes $i$ and $j$ and $T$ is a reference time here chosen as 1 year. On the basis of both $\{a_{ij}\}$ and $\{p_{ij}\}$ (or the complementary failure probability matrix $\{q_{ij}\}$), the matrix of the most reliable path lengths $\{d_{ij}\}$ can be computed as [2]:

$$d_{ij} = \min_{\gamma} \left\{ \prod_{mn \in \gamma} p_{mn} \right\} = \min_{\gamma} \left\{ \prod_{mn \in \gamma} (1 - q_{mn}) \right\},$$

where the minimization is done with respect to all paths $\gamma$ linking nodes $i$ and $j$ and the product extends to all the edges of each of these paths. Note that $1 \leq d_{ij} \leq \infty$, the lower value corresponding to the existence of a perfectly reliable path connecting $i$ and $j$ (i.e. $p_{mn} = 1, q_{mn} = 0 \ \forall mn \in ij$) and the upper value corresponding to the situation of no paths connecting $i$ and $j$ (i.e. $p_{mn} = 0, q_{mn} = 1$).

A power flow reliability efficiency between two nodes $i$ and $j$ can then be defined as inversely proportional to the reliability distance $d_{ij}$ of the most reliable path linking them. Thus, the network is characterized also by the matrix $\{e_{ij}\}$ whose entry is the reliability efficiency in the power flow transmission between nodes $i$ and $j$ and an average reliability efficiency can be computed for the transmission network system $G$:

$$E_r(G) = \frac{\sum_{i \neq j \in G} e_{ij}}{N(N-1)}$$

Similarly to the characteristic path length $L$, this quantity defines the network connection characteristics on a global scale, with the difference that it also accounts for the reliability of the edges in providing the power transmission. Whereas $L$ takes into account only the links from one node to another in the sequential paths along the network, the reliability efficiency measure (8) retains also the information about the reliability of these paths which concurrently give contribution to the power flow, akin to a parallel system [3]. Since $e_{ij} = 1$ when there is at least one infallible path $\gamma$ in the graph which connects nodes $i$ and $j$ through a sequence of non-failing edges, $E_r(G)$ is equal to one in case of a non-failing, perfectly connected network. By similarity with the topological efficiency indicator introduced in Section 3.1, the local properties of the graph $G$ can be quantified by specializing the definition of the average reliability efficiency (8) on the subgraph $G_i$ of the neighbors of each node $i$ in the network (4), averaged over all nodes in the network to give the network local reliability efficiency $E_{rlc}(G)$ defined as in (5):

$$E_{rlc}(G) = \frac{1}{N} \sum_{i=1}^{N} E_r(G_i)$$

This parameter reveals how much the network is fault tolerant in that it shows how reliable the power transmission remains among the first neighbors of $i$ when $i$ is removed.

4.2. Reliability efficiency analysis of the IEEE 14 BUS transmission network

Table 1 provides the failure rates of the components of the transmission network, as inferred from literature data [8]. Assuming that the arcs failure rates remain constant during the mission time of 1 yr, it is immediate to compute the reliabilities $p_{ij}$ and thus the average global and local connection reliability efficiencies: $E_r = 0.3104; E_{rlc} = 0.1864$.

These values confirm the small-world connection properties of the network which are retained even when accounting for the probabilities of failure of its components.
Table 1. Failure rates for the arcs

<table>
<thead>
<tr>
<th>From BUS</th>
<th>To BUS</th>
<th>Failure rate (occ/yr)</th>
<th>Equipment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>1.0858</td>
<td>Transmission line 132 kV</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>1.0858</td>
<td>Transmission line 132 kV</td>
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<tr>
<td>2</td>
<td>3</td>
<td>1.0858</td>
<td>Transmission line 132 kV</td>
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<tr>
<td>2</td>
<td>4</td>
<td>1.0858</td>
<td>Transmission line 132 kV</td>
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<td>2</td>
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<td>Transmission line 132 kV</td>
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<tr>
<td>3</td>
<td>4</td>
<td>1.0858</td>
<td>Transmission line 132 kV</td>
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<tr>
<td>4</td>
<td>5</td>
<td>1.0858</td>
<td>Transmission line 132 kV</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>0.0105</td>
<td>Transformer 132/230 kV</td>
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<tr>
<td>4</td>
<td>9</td>
<td>0.0105</td>
<td>Transformer 132/230 kV</td>
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<tr>
<td>5</td>
<td>6</td>
<td>0.0105</td>
<td>Transformer 132/230 kV</td>
</tr>
<tr>
<td>6</td>
<td>11</td>
<td>0.5429</td>
<td>Transmission line 230 kV</td>
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<tr>
<td>6</td>
<td>12</td>
<td>0.5429</td>
<td>Transmission line 230 kV</td>
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<tr>
<td>6</td>
<td>13</td>
<td>0.5429</td>
<td>Transmission line 230 kV</td>
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<td>7</td>
<td>8</td>
<td>0.0105</td>
<td>Transformer 132/230 kV</td>
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<td>7</td>
<td>9</td>
<td>0.0105</td>
<td>Transformer 132/230 kV</td>
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<td>10</td>
<td>0.5429</td>
<td>Transmission line 230 kV</td>
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<td>9</td>
<td>14</td>
<td>0.5429</td>
<td>Transmission line 230 kV</td>
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<td>10</td>
<td>11</td>
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<td>12</td>
<td>13</td>
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<td>13</td>
<td>14</td>
<td>0.5429</td>
<td>Transmission line 230 kV</td>
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</table>

4.3. Validation by Monte Carlo efficiency estimation

To validate the reliability efficiency values obtained, a Monte Carlo estimation of the network efficiencies (3) and (5) has been performed, accounting for the arcs reliability $p_{ij}$. A total of $10^5$ network configurations have been sampled from the $p_{ij}$'s. For each sampled configuration, the global and local topological efficiencies have been calculated according to equations (3) and (5). The results obtained are: $E_{rMC} = 0.2801 \pm 0.0010$; $E_{rlMC} = 0.1434 \pm 0.0014$.

These estimates compare well with the values of average global and local connection reliability efficiencies computed by equations (8) and (9). This provides physical validation to such indicators which can then be fruitfully used for the analysis of networks of realistic dimensions for which a Monte Carlo estimation procedure would be impractical for the excessive computing times involved.

4.4. Robustness analysis

The analysis of the robustness of the transmission system to random failures has been performed by uniform random removal of a number of arcs. Where necessary, the random disconnection procedure was repeated several times to obtain the necessary statistical accuracy. For brevity’s sake only the results on the global and local efficiencies are reported.

With respect to the variation in global efficiency, Figure 3 shows the expected monotonic decrease for both topological and reliability curves. The reduction in global reliability efficiency is slightly higher for the global efficiency; the reliability curve starts and ends with the same values as the topological curve, but during the removal of the nodes it gets beyond the topological curve because of the failure rates of the equipment.
Figure 3. Global efficiency relative variation as a function of the number of removed arcs

![Global efficiency relative variation](image)

The relative variation of the local efficiency presented in Figure 4 is quite similar for both topological and reliability connections. The curve of the local efficiency reliability variation follows closely the topological one from above due to the values of the arcs reliability parameters. Anyway, at the local level the difference between the topological and reliability efficiency is mild. This indicates that for the small and sparse subgraphs of each node, for which there are usually no shortest paths involving more than one edge, the topology and the reliability characteristics bring the same information to the relative variation of local efficiency, i.e. local subgraphs will show differences in the absolute value of local efficiency between topological and reliability analysis but they will not influence its relative variation.

Figure 4. Local efficiency relative variation as a function of the number of removed arcs

![Local efficiency relative variation](image)

4.5. Resilience analysis

For brevity’s sake only the results on the global and local efficiencies are reported. Figures 5 and 6 show the results of the resilience analysis against directed attacks performed on the transmission system by removing one node at a time. The removal of a node implies the removal of all the arcs incident onto that node.

Figure 5 presents the relative variation of the global efficiency of the connection, from both topological and reliability points of view. The two curves present a quite similar behaviour as a function of the removed node, having similar trend, but different in intensities due to the reliability values of the network components.

Table 2 shows all the nodes of the network ranked according to the relative variation in the topological (line two) and reliability (line three) global efficiency caused by their removal. Node 4, which from a topological point of view is the most connected, drops to the third place (-24.41%) in the reliability
global efficiency rank, due to the fact that node 7 and 9 have very low connections’ failure rates. Notice that node 7 ranks second (-23.04%) in the topological analysis, although only three edges depart from it, because its removal implies also the removal of node 8 and of all the shortest paths in the $d_{ij}$ matrix originating from it. Node 2 drops in rank from the sixth topological to the twelfth reliability position, due to the fact that the four incident arcs are all characterized by the largest values of the failure rates. Finally, node 8, which is the last one from a topological point of view (-10.2%), gains the tenth position in the reliability global efficiency rank (-13.85%), since its only connection to the rest of the network is a highly reliable one.

Table 2. Topological and reliability global efficiency ranking. In the second and third line the network nodes are ranked according to the relative variation of topological and reliability global efficiency caused by their removal

<table>
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<tr>
<th>Rank</th>
<th>1</th>
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<tr>
<td>$\Delta E/E$</td>
<td>4</td>
<td>7</td>
<td>9</td>
<td>6</td>
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<td>2</td>
<td>13</td>
<td>14</td>
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<td>1</td>
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<td>8</td>
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<tr>
<td>$\Delta E_r/E_r$</td>
<td>7</td>
<td>9</td>
<td>4</td>
<td>6</td>
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<td>12</td>
<td>2</td>
<td>3</td>
<td>1</td>
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</tbody>
</table>

Figure 5. Global efficiency relative variation as a function of removed node

The relative variation of the local efficiency plotted in Figure 6 has a similar behaviour from both the topological and reliability points of view. At local scale, node 2 has the best connected subgraph due to the connections between its first neighbors, so that its removal has the largest relative variation (57.5%) in the topological case. Notice the different position of node 2 in the previous topological rank: this is due to the different information conveyed by the local efficiency indicator. Analogously to the global case, Table 3 shows all the nodes of the network ranked according to the relative variation in the topological (line two) and reliability (line three) local efficiency. Node 5 presents the largest position drop when considering the reliability rank: this is due to the fact that its local subgraph (nodes 1, 2, 4, 6) is characterized by two little reliable connections (failure rates 1.0858 occ/yr). On the contrary, node 9 gains three positions since the edge between nodes 4 and 7 (its local subgraph) is highly reliable (failure rate $\lambda_{47} = 0.0105$ occ/yr).

Table 3. Topological and reliability local efficiency ranking. In the second and third line the network nodes are ranked according to the relative variation of topological and reliability local efficiency caused by their removal

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<tr>
<th>Rank</th>
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<tbody>
<tr>
<td>$\Delta E_{loc}/E_{loc}$</td>
<td>2</td>
<td>4</td>
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<td>12,13</td>
<td>6</td>
<td>1</td>
<td>3</td>
<td>9</td>
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<td>10,11</td>
<td>8</td>
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<td>$\Delta E_{rel,loc}/E_{rel,loc}$</td>
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<td>2</td>
<td>12,13</td>
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<td>3</td>
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5. CONCLUSIONS

In this paper, a systematic procedure of analysis of a power transmission system has been carried out from both the topological and reliability points of view. The IEEE 14 BUS was taken as case study. Each equipment of the system was transposed into a node or edge of the representative network and various topological and efficiency indicators were computed to characterize its structure and reliability performance. The results obtained have been compared with those obtained for a random network with the same number of arcs and nodes, demonstrating that the 14 BUS network bears the favourable properties of small world networks, i.e. good global and local connectivity. A Monte Carlo validation of the significance of these indicators was also performed.

Finally, a crude procedure of analysis of the network resilience and robustness was carried out by analyzing the effects on the network topological and reliability properties of removal of nodes and arcs, one at a time. It was shown that taking into account the reliability characteristics of the network elements is relevant as their effect on the system robustness and resilience characteristics may differ from that due to the purely topological position in the system. This allows properly identifying the critical points of the network so as to manage and protect its vulnerability for the optimal functionality of the network.

Acknowledgements

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References

[6] The network visualizations were done using the Pajek program for large network analysis: http://vlado.fmf.uni-lj.si/pub/networks/pajek/